ii) Galaxy clusters are more representative of the overall structure of the Univ.

- consider N galaxies all bound together by gravity

for the i^{th} galaxy:
$$\ddot{\vec{x}}_i = G \sum_{j \neq i} m_j \frac{\vec{x}_j - \vec{x}_i}{|x_j - x_i|^3}$$

The grav. P.E. of the system of N galaxies is

$$W = -\frac{G}{2} \sum_{\substack{i,j \\ j \neq i}} \frac{m_i m_j}{|\vec{x}_j - \vec{x}_i|}$$
($\frac{1}{2}$ makes sure we remove the double-counting)

$$= -\alpha \frac{GM^2}{r_h}$$
where $M = \sum m_i$
r_h = half-mass rad.
$\alpha \simeq 0.4$ (density profile)

The KE associated w/ the motions of the galaxies $K = \frac{1}{2} \sum_i m_i |\dot{\vec{x}}_i|^2$ or $K = \frac{1}{2} M \langle v^2 \rangle$

where $\langle v^2 \rangle = \frac{1}{M} \sum m_i |\dot{x}_i|^2$ is the mean sq. velocity

Define I, the moment of inertia of the cluster

$$I = \sum_i m_i |\vec{x}_i|^2$$

$$\ddot{I} = 2 \sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i + \dot{\vec{x}}_i \cdot \dot{\vec{x}}_i)$$

$$= 2\sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) + 4K$$

and $\sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) = G \sum_{\substack{i,j \\ i \neq j}} m_i m_j \frac{\vec{x}_i \cdot (\vec{x}_j - \vec{x}_i)}{|\vec{x}_j - \vec{x}_i|^3}$

or $\sum_j m_j (\vec{x}_j \cdot \ddot{\vec{x}}_j) = G \sum_{\substack{j,i \\ j \neq i}} m_j m_i \frac{\vec{x}_j \cdot (\vec{x}_i - \vec{x}_j)}{|\vec{x}_j - \vec{x}_i|^3}$

These 2 are equal, so

$$\sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) = \frac{1}{2}\left[\sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) + \sum_j m_j (\vec{x}_j \cdot \ddot{\vec{x}}_j)\right]$$

$$= -\frac{G}{2} \sum_{\substack{i,j \\ i \neq j}} \frac{m_i m_j}{|\vec{x}_j - \vec{x}_i|} = W$$

or $\boxed{\ddot{I} = 2W + 4K}$ Virial Theorem

In steady state, $I = \text{const.}$

$$0 = W + 2K$$

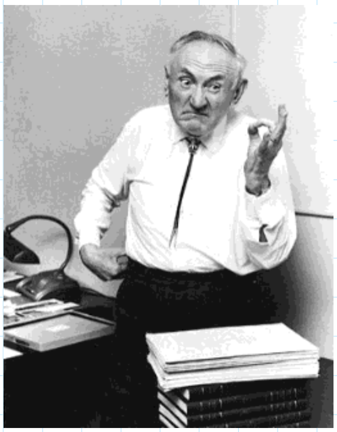
$$\text{or } K = -\frac{W}{2}$$

For a self-gravitating system in steady state, the KE is $-\frac{1}{2} \times$ the grav. PE

$$\text{so } \frac{1}{2} M \langle v^2 \rangle = \frac{\alpha}{2} \frac{GM^2}{r_h}$$

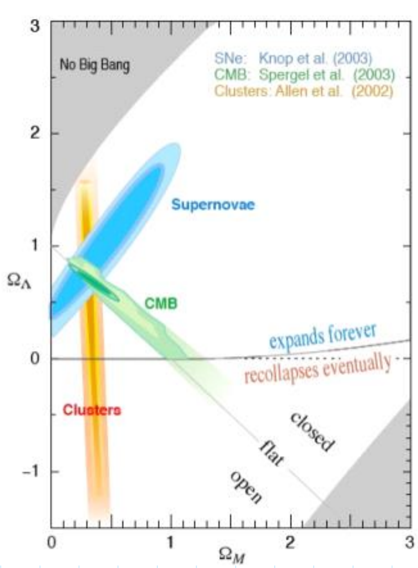
$$M = \frac{\langle v^2 \rangle r_h}{}$$

Measuring $\langle v^2 \rangle$ and r_h [$\propto G$] are tricky but can be done. Find cluster masses ~10x more than the gas mass $\therefore$ $\Omega_{m,\text{clusters}} \simeq 0.2$



Fritz Zwick was the 1st to point out that cluster galaxies were moving too fast for the amount of luminous matter

Bullet cluster
red = gas
blue = gravity (from lensing)



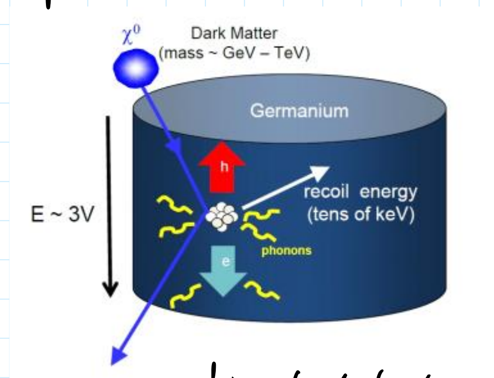
By combining the cluster results w/ SNe & CMB (still to come) get best constraints on $\Omega_{m,0} \simeq 0.3$ w/ $\Omega_{\text{baryon}} \simeq 0.04$

Most of the matter in the Univ. is not baryons and only interacts grav. $\rightarrow$ DARK MATTER

What is Dark Matter?

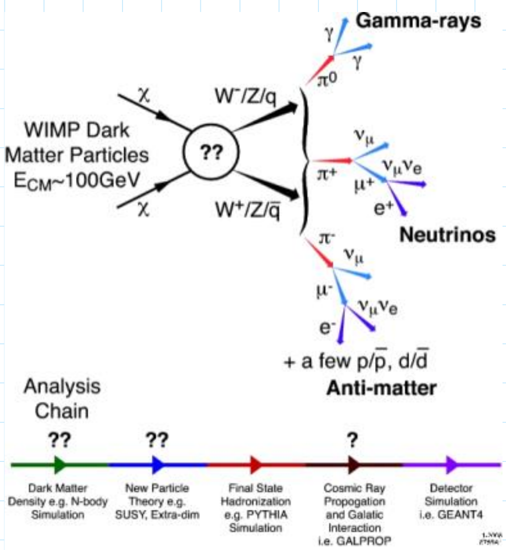
- neutrinos? NOPE Its bad for structure formation. If too heavy, bad for particle physics

- supersymmetric particles, axions, other WIMPs. Need to find them



direct detection

annihilation lines from astrophysical sources



- MACHOs - detected but not enough of them

- something else?