Phys 4247 Midterm #2 - Solutions

1. Surface brightness is the flux divided by the (angular size)^2:

$$I = \frac{F}{\theta^2}$$ where F = bolometric flux ; θ = angular size

But $L = 4\pi d_L^2 F$ where d_L is the luminosity distance

and $\theta = \frac{D}{d_A}$ where d_A is the angular diameter distance

$$\therefore I = \frac{L}{4\pi d_L^2 D^2} d_A^2 = \frac{L}{4\pi D^2}\left(\frac{d_A}{d_L}\right)^2$$

but $\frac{d_A}{d_L} = \frac{1}{(1+z)^2}$, so $I = \frac{L}{4\pi D^2}\frac{1}{(1+z)^4}$

2. Y is defined as mass fraction of ^4He to all baryon mass. All baryon mass $= (n_n + n_p)m_p$ where n_n and n_p are number (densities) of neutrons and protons and m_p = proton mass

Max. possible value of the primordial Y would occur if all neutrons end up in ^4He. Since there are 2 neutrons in each ^4He, the max. number density of ^4He is $n_n/2$. But each ^4He weighs about $4m_p$, so max. mass density of $^4He = \frac{n_n}{2} \times 4m_p = 2n_n m_p$. $\therefore Y_{max} = \frac{2n_n m_p}{(n_n+n_p)m_p} = \frac{2\frac{n_n}{n_p}}{\frac{n_n}{n_p}+1} = \frac{2f}{1+f}$ where $f = \frac{n_n}{n_p}$

3. Infationary expansion is defined as $\ddot{a} > 0$.

$\therefore$ If $a \propto t^m$, then inflationary expansion occurs if $\dot{a} \propto m t^{m-1}$

$\rightarrow \ddot{a} \propto m(m-1)t^{m-2} > 0$

$\rightarrow$ <u>$m > 1$</u>